38th Annual SANUM Conference

University of the Witwatersrand

Programme Schedule

Contact People:

- Charis Harley
  charis.harley@wits.ac.za
- Precious Shabalala
  precious.shabalala@wits.ac.za
- Byron Jacobs
  byron.jacobs@wits.ac.za
- Michael Mitchley
  michael.mitchley@wits.ac.za

Venues:

All contributed and Plenary talks will take place at the Wits Club in either the Middle Room or the East Room.

The opening and closing addresses will take place at the Wits Club.

Tea and Lunch:

- Tea Breaks will be held in the Bar/Reception area outside of the conference venues.
- Lunch Breaks will be held in the Wits Club restaurant.
- The conference dinner will be held at the Wits Club restaurant at 18:30 for 19:00.

Welcome Address

The School of Computational and Applied Mathematics and the Faculty of Science at the University of the Witwatersrand, Johannesburg would like to welcome the plenary speakers and participants to the 38th Annual South African Symposium on Numerical and Applied Mathematics.

The importance of mathematical sciences to the development of South Africa has been highlighted in the public and private sector. The Department of Science and Technology and the National Research Foundation of South Africa through an intensive peer-review process have awarded the University of the Witwatersrand a Centre of Excellence in the Mathematical Sciences. One of the main purposes of the Centre is to develop the next generation of mathematicians and applied mathematicians with the ability to make a significant contribution to both industry and academia.

SANUM is a key component of this human capacity development as it brings together, not only local, but international experts in the fields of numerical methods, analysis and computation. We hope that your time with us will be intellectually stimulating and that your interactions with each other will assist in enhancing your research.

E. Momoniat
Head of School
Chair of Mathematical Sciences

List of Participants

Dr. Richard Akinola	*University of Cape Town*
Dr. Ademola Badmus	*Nigerian Defence Academy Kaduna*
Prof Tanmay Basak	*Indian Institute of Technology Madras*
Ms. Madelein Basson	*University of Pretoria*
Dr. Simon Childs	*University of Free state*
Prof Gordon Cooper	*University of the Witwatersrand*
Ms. Sonja du Toit	*University of Pretoria*
Dr. Inger Fabris-Rotelli	*University of Pretoria*
Dr. Adewunmi Fareo	*University of the Witwatersrand*
Mr. Hagos Gidey	*University of Pretoria*
Dr. Nick Hale	*Stellenbosch University*
Dr . Charis Harley	*University of the Witwatersrand*
Mr. Andries Haywood	*University of Pretoria*
Prof Bruce Henry	*The University Of New South Wales*
Mr. Rhameez Herbst	*University of the Witwatersrand*
Prof Willy Hereman	*Colorado School of Mines*
Mr. Fabrice Iandu	*Decratic Republic of Congo*
Mr. Byron Jacobs	*University of the Witwatersrand*
Mr. Olaoluwa Jejeniwa	*University of Pretoria*
Mr. Rasaq Kareem	*Lagos State Polytechnic*
Ms. Rahab Kgatle	*University of the Witwatersrand*
Prof John King	*University of Nottingham*
Mr. Pradeep Kumar	*University of the Witwatersrand*
Prof Fazal Mahomed	*University of the Witwatersrand*
Prof David Mason	*University of the Witwatersrand*
Mr. Rofhiwa Matumba	*University of Venda*
Mr. Michael Mitchley	*University of the Witwatersrand*
Mr. Umaru Mohammed	*Unaffiliated*
Prof Joel Moitsheki	*University of the Witwatersrand*
Ms. Mapule Mokae	*Unaffiliated*
Dr. Gift Muchatibaya	*Cape Peninsula University Of Technology*
Ms. Winifred Mutuku	*Cape Peninsula University Of Technology*
Dr. Imran Naeem	*Lahore University of Management Sciences*
Mr. Ndivhuwo Ndou	*University of Venda*
Prof Jan Nordström	*Linköping University*
Mr. Jeffrey Ochoche	*University of Agriculture, Makurdi*
Ms. Sonali Parboo	*University of the Witwatersrand*
Prof Lothar Reichel	*Kent State University*
Mr. Abdu Sagir	*Hassan Usman Katsina Polytechnic, Nigeria*
Ms. Belinda Stapelberg	*University of Pretoria*
Dr. Alna van der Merwe	*Auckland University of Technology*
Prof Nic van Rensburg	*University of Pretoria*
Prof André Weideman	*Stellenbosch University*
Mr. Matthew Woolway	*University of the Witwatersrand*
Ms. Ayobami Zamurat	*Kaduna Polytechnic , Kaduna*

Plenary Abstracts

Finite Element Based Simulation Studies on Various Heating Applications

Tanmay Basak

Finite element method has been found to be a powerful technique for analyzing flow and thermal characteristics involving various heating applications such as natural and mixed convection and radiation assisted processing. Material processing and thermal management have received significant attention and in this context, this lecture is aimed to address thermal management for convection systems and various material processing scenarios during microwave assisted processing. Galerkin finite element method has been used to solve thermal and flow characteristics. Bi-quadratic elements are found to give converged solution with lesser mesh sizes compared to standard control volume or finite difference based solution strategies. Energy balance coupled with momentum balance equations are solved using Penalty finite element method. Finite element based analysis has been carried out for multiple solutions and bifurcations of steady natural convections in cavities. A generic spectrum of multiple symmetry breaking flow and thermal characteristics has been obtained based on robust finite element in house code. Further, thermal management has been analyzed via finite element post processing based on streamfunction, heatfunction, and entropy generation. Finite element method has been found to be extremely powerful for heat flow distributions via trajectories of heatlines (represented via heatfunctions). Major challenge on evaluation of heatfunctions is based on solutions of Poisson equations with integral boundary conditions within any irregular or non-square domain. Efficiency of a thermal convection system is quantified via estimation of exergy loss based on minimization of entropy generation due to heat transfer and fluid flow irreversibilities. Finite element based basis functions are also found to be robust on evaluation of non-linear derivatives involving both fluid flow and thermal irreversibilities especially in irregular domains. Case studies have been demonstrated for various domains involving triangular, trapezoidal and complex cavities with various heating sources at walls. A few case studies are also shown for natural convection involving various distributed heating patterns. Efficiency of processing and thermal management have been demonstrated for above cases via heatlines and entropy generation for various natural and mixed convection problems. Finite element simulations are also demonstrated for microwave assisted volumetric thermal processing for various samples. A major challenge on finite element modeling on microwave transport processes involves integro-differential boundary conditions with Helmholtz equations of complex electric field in irregular domains. A few interesting results based on resonances of power absorption and counter-intuitive thermal characteristics during microwave assisted thermal processing have been demonstrated.

Chebfun and Related Research: $\{\textbf{software}\} \cap \{\textbf{research}\} \neq \emptyset$!

Nick Hale

Chebfun:

Chebfun is an open-source project written in object-oriented Matlab [1]. Originating in 2004, one of its principal aims was to explore a new paradigm of computing that **feels** symbolic but maintains the **speed** of numerics. Now, 10 years later and approaching the release of its 5th version, the software contains state of the art routines for 1 and 2 dimensional function approximation, quadrature, numerical solution of ODEs, and much more. In this talk I hope to not only give a brief introduction to the software and demo some of its recent features, but give a couple of examples (time-permitting) of my own research which arose out of the project, hopefully dispelling the myth that software development and research are mutually exclusive.

Rectangular projection for spectral collocation:

Spectral collocation (or pseudospectral) methods are based on the idea of interpolating the unknown solution of a differential equation by a polynomial, the derivative of which is known in closed form. This leads to a *differentiation matrix* which, once sufficient boundary conditions have been imposed, can be inverted to find an approximation to the underlying solution. The standard "boundary bordering" approach removes one or more rows from the matrix in order to apply these conditions, but since Chebfun's ODE functionality works as a black box, we found it was not always immediately obvious which rows were to be removed. This lead to the derivation of a new technique, which we refer to as "rectangular projection", where the rows of the matrix are projected on to a lower degree polynomial, rather than discarded [2].

A fast Chebyshev-Legendre transform:
The mathematical basis of the Chebfun system is interpolation by Chebyshev polynomials. However, sometimes a Legendre series representation of a function can be much more useful, for example in computing best L^2 approximations, or convolutions [3]. Existing methods for computing these transforms were either slow, unstable, or unsuitable for Matlab implementation [4], so (based on some earlier work for the fast computation of Gauss–Legendre quadrature nodes and weights [5]) we derived a new fast and stable transform based upon asymptotic expansions and the FFT [6].

[1] Chebfun, `http://www.chebfun.org/`.

[2] T.A. Driscoll and N. Hale, *Resampling methods for boundary conditions in spectral collocation*, (In prep.).

[3] N. Hale and A. Townsend, *An algorithm for the convolution of Legendre series*, (To appear in SISC).

[4] B.K. Alpert and V. Rokhlin, *A fast algorithm for the evaluation of Legendre expansions*, SISC, 12.1, pp. 158–179, 1991.

[5] N. Hale and A. Townsend, *Fast and accurate computation of Gauss–Legendre and Gauss–Jacobi quadrature nodes and weights*, SISC, 35.2, pp. A652–A674, 2013.

[6] N. Hale and A. Townsend, *A fast, simple, and stable Chebyshev-Legendre transform using an asymptotic formula*, SISC, 36.1, pp. A148–A167, 2014.

Stochastic Models for Diffusive Transport with Traps, Reactions and Forces

Bruce Henry

A major theoretical challenge of the past decade in anomalous diffusion research has been to derive evolution equations for subdiffusive transport taking into account further complications from force fields and reactions. Starting with the continuous time random walk as the underlying stochastic process we derive the generalized master equations for an ensemble of particles undergoing reactions whilst being subject to an external force field. From the generalized master equation we show reductions to special cases, including; the fractional reaction diffusion equation for subdiffusion with nonlinear reaction kinetics and the fractional Fokker-Planck equation for subdiffusion in a space- and time- dependent force field. Some applications are discussed.

Symbolic Computation of Conservation Laws of Nonlinear Partial Differential Equations

Willy Hereman

A method will be presented for the symbolic computation of conservation laws of nonlinear partial differential equations (PDEs) involving multiple space variables and time. Using the scaling symmetries of the PDE, the conserved densities are constructed as linear combinations of scaling homogeneous terms with undetermined coefficients. The variational derivative is used to compute the undetermined coefficients. The homotopy operator is used to invert the divergence operator, leading to the analytic expression of the flux vector. The method is algorithmic and has been implemented in the syntax of the computer algebra system Mathematica. The software is being used to compute conservation laws of nonlinear PDEs occurring in the applied sciences and engineering. The software package will be demonstrated for PDEs that model shallow water waves, ion-acoustic waves in plasmas, sound waves in nonlinear media, and transonic gas flow. The featured equations include the Korteweg-de Vries, Kadomtsev-Petviashvili, Zakharov-Kuznetsov, and Khoklov-Zabolotskaya equations. A similar approach can be used to find generalized symmetries, recursion operators, and Lax pair operators for nonlinear completely integrable PDEs and differential-difference equations.

Self-Similar Behaviour in Nonlinear Diffusion

John King

The diffusion equation with a diffusivity that is a power of the dependent variable is widely studied, both for its large number of physical and biological applications and as a paradigm problem in the theory of nonlinear partial differential equations. A number of initial boundary problems will be analysed, demonstrating the significance of a number of distinct classes of self-similarity and clarifying the role of the corresponding linear equation (i.e. the heat equation) as a special case.

High Order Finite Difference Approximations of Multi-physics Problems

Jan Nordström

Great progress have been made for high order finite difference methods during the last 20 years. It is now rather straightforward to construct stable and high order accurate multi-block finite difference schemes in a building-block-like manner. In this talk we will discuss new developments, new extensions, new fields of applications, share experience on the full scale use of the technique, and speculate about future trends. We present the theory by using simple model problems, and exemplify with complex multi-physics applications.

First we walk through the basic technique for a simple model problem where all the ingredients such as well-posedness, summation-by-parts operators, penalty techniques for boundary and interface conditions, and the energy method are explained. Next, we discuss the application of the theory to coupling techniques for conjugate heat transfer, earthquake simulations, fluid- solid interaction and simulation of potential propagation in neuronal branches. Finally we discuss new time-integration procedures, new boundary conditions for the Navier-Stokes equations and a new variance reduction technique for stochastic partial differential equations.

Network Analysis via Gauss-Type Quadrature and Partial Spectral Factorization

Lothar Reichel

Networks arise in many applications. It is often of interest to be able to identify the most important nodes of a network or to determine the ease of traveling between them. We are interested in carrying out these tasks for large undirected and directed networks. Many quantities of interest can be determined by computing certain matrix functionals. We discuss how for directed and undirected graphs a few steps of the block Lanczos method, in combination with block Gauss and block anti-Gauss quadrature rules, can be applied to provide estimates of upper and lower bounds for quantities of interest. Also approaches that are based on partial spectral factorization or partial singular value decomposition in combination with Gauss-type quadrature for bounding pertinent quantities will be described. Computed examples illustrate the performance of the approaches discussed.

Abstracts

Hybrid Implicit Block Algorithms for Improved Performance in the Solution of First Order Initial Value Problems

Ademola Badmus

We propose 8- point hybrid implicit linear multistep block method for with grids and off grid points of for the solution of first order initial value problems of the form . The main integrator in this method is of order 10 and the rest seven integrators are of uniform order 9. All the schemes derived to form our block method come from a single Continuous formulation. The implementation is demonstrated with linear and non linear problems to ascertain their level of accuracy.

Galerkin Finite Element Approximation of General Linear Second Order Hyperbolic Equations with General Damping

Madelein Basson

We derive error estimates for the Galerkin approximation of a general linear second order hyperbolic partial differential equation with general damping (which includes boundary damping). The results can be applied to a variety of cases e.g. vibrating systems of linked elastic bodies. The results generalize the work of Dupont (1973). Splitting the proofs for the semidiscrete and fully discrete cases not only simplifies the proofs but less restrictive regularity assumptions are required.

The Finite Element Implementation of a K.P.P. Equation for the Simulation of Tsetse Control Measures in the Vicinity of a Game Reserve

Simon Childs

An equation, strongly reminiscent of Fisher's equation, is used to model the response of tsetse populations to proposed control measures in the vicinity of a game reserve. The model assumes movement is by diffusion and that growth is logistic. This logistic growth is dependent on an historical population, in contrast to Fisher's equation which bases it on the present population. The model therefore takes into account the fact that new additions to the adult fly population are, in actual fact, the descendents of a population which existed one puparial duration ago, furthermore, that this puparial duration is temperature dependent. Artificially imposed mortality is modelled as a proportion at a constant rate. Fisher's equation is also solved as a formality.

The temporary imposition of a 2 % day^{-1} mortality everywhere outside the reserve for a period of 2 years will have no lasting effect on the influence of the reserve on either the Glossina austeni or the G. brevipalpis populations, although it certainly will eradicate tsetse from poor habitat, outside the reserve. A 5 km-wide barrier with a minimum mortality of 4 % day^{-1}, throughout, will succeed in isolating a worst-case, G. austeni population and its associated trypanosomiasis from the surrounding areas. A more optimistic estimate of its mobility suggests a mortality of 2 % day^{-1} will suffice. For a given target-related mortality, more mobile species are found to be more vulnerable to eradication than more sedentary species, while the opposite is true for containment.

Determining the Depth and Location of Dykes from Aeromagnetic Data

Gordon Cooper

The ratio of the analytic signal amplitudes of orders 1 and 2 of the magnetic anomaly from a dyke is shown to give the distance r to the dyke. Local minima of r give the depth to the dyke, and the position of these minima gives its location. Because of the use of the analytic signal amplitude, the depth to the top of the dyke is almost unaffected by the dykes magnetisation vector.

Finite Element Analysis of the Vibration of a Vertical Slender Structure on a Resilient Seating

Sonja du Toit

Unwanted vibrations often occur in mechanical structures. Because of their inherent low damping, free-standing welded steel structures are prone to oscillate in the wind. One method of artificially increasing the damping is to mount the chimney on a resilient foundation incorporating bearing pads made of high-damping material. In this talk we consider application of the Finite element method for a hybrid Rayleigh beam model.

Compressed Sensing

Inger Fabris-Rotelli

Compressed sensing allows for 'sensing' less information but gaining more than originally thought possible. We provide an overview of this fascinating new technique and what it provides for the field of image processing.

Approximate Solutions to the Problem of a Pre-existing Fracture Driven by a Power-law Fluid

Gideon Fareo

In this paper we derive approximate solutions to the problem of a pre-existing two- dimensional fracture with non-zero initial length driven by the injection of an in- compressible non-Newtonian fluid of power-law rheology. The pre-existing fracture propagates in a permeable rock medium. The fluid flow in the fracture is considered laminar. The PKN approximation in which the fluid pressure is proportional to the width of the fracture is used. By considering the ratio of the fluid velocity to the speed of propagation of the fracture tip, approximate analytical solutions are derived.

Time-Splitting Procedures for the Numerical Solution of the 2D Advection-Diffusion Equation

Hagos Gidey

Spectral analysis of the dispersive and dissipative properties of two time-splitting procedures, namely, locally one-dimensional (LOD) Lax-Wendroff and LOD (1,5) for the numerical experiment of the 2D advection-diffusion equation is performed. We solved a 2D numerical experiment described by an advection-diffusion partial differential equation with specified initial and boundary conditions for which the exact solution is known. We perform a stability analysis of the two time-splitting procedures. Some errors are computed, namely, the error rate with respect to the L_1 norm and dispersion and dissipation errors. Lastly, an optimization technique is used to find the optimal value of temporal step size that minimizes the dispersion error for both schemes when the spatial step is chosen as 0.025, and this is validated by numerical experiments.

Particle Occlusion in Gold Nanoparticles

Andries Haywood

Nanotechnology is a fast growing research field with its main applications in medical and material sciences. In all applications size and size distribution of nanoparticles is of primary concern. However, particle occlusion is most often an unwanted phenomenon occurring during the image analysis. We present a semi-automated Bayesian technique to combat this occlusion to provide improved measurements and discuss various possible further optimised techniques.

Numerical Simulation of Isothermal Gravitational Collapse

Rhameez Herbst

Gravitational Collapse of Isothermal Spheres is investigated under various starting conditions. The appearance of shocks during collapse depends not only on the starting conditions but also the manner in which the model is set up. We consider the three main types of initial condition: uniform density (investigated by Larson and Penston), the singular isothermal sphere of Shu and the Bonnor-Ebert sphere investigated by Foster. We compare each case with and without a low density environment. Finally we examine the late time results and compare them to those of classical Bondi accretion.

A New Grünwald-Letnikov Derivative Derived From a Second-Order Scheme

Byron Jacobs

A novel derivation of a second-order accurate Grünwald-Letnikov type approximation to the fractional derivative of a function is presented. This scheme is shown to be second-order accurate under certain modifications to account for poor accuracy in approximating the asymptotic behavior near the lower limit of differentiation. Some example functions are chosen and numerical results presented to illustrate the efficacy of this new method over some other popular choices for discretizing fractional derivatives.

Numerical Solutions of stiff Initial Value Problems in ODEs Using Modified Extended Backward Differentiation Formula

Rasaq Kareem

This paper seeks numerical solutions to a stiff initial value problem in ordinary differential equations (ODEs) using a modified extended backward differentiation formula. Some implicit schemes were developed based on the linear multi-step methods. Stiff initial value problems were solved using the various stages of the derived modified extended backward differentiation formula and the results obtained at different stages are compared with each other in determining which of the schemes is more accurate and efficient in providing solutions to stiff initial value problems in ODEs when compared with the exact solution.

Propagation of a Linear Hydraulic Fracture with Tortuosity

Rahab Kgatle

The propagation of a pre-existing hydraulic fracture with tortuosity in the fluid flow is investigated. The tortuosity is caused by the roughness of the crack walls and by areas of contact between asperities (deformations) on opposite crack walls. The normal stress at the crack walls is distributed between the fluid pressure and the contact areas of touching asperities. The tortuous fracture is replaced by a symmetric open fracture without asperities with a modified Reynolds flow law and modified stress in the fracture. For a partially open tortuous fracture the linear crack law is used in which the half-width is related to the effective pressure by a piecewise linear law. The Perkins-Kern-Nordgren approximation is made in which the normal stress at the crack walls is proportional to the half-width of the symmetric model fracture. A Lie point symmetry analysis is used to formulate a group invariant solution for the length, volume and half-width of the pre-existing fracture. Exact analytical solutions are derived for fractures with constant volume and constant speed of propagation and a numerical solution is developed for general operating conditions at the fracture entry. It is found that tortuosity can remove the singularity in the spatial gradient of the half-width at the fracture tip of the model fracture and that the length of the hydraulic fracture becomes less dependent on the operating conditions at the fracture entry as the tortuosity increases. From the numerical solution it is found that the fluid velocity averaged over the width of the model fracture increases approximately linearly along the fracture and this observation is used to derive an approximate analytical solution for the length and half-width which agrees well with the numerical solution.

In Silico Mathematical Analysis and Programming for the Performance Interpretation of Drug Eluting Assemblies: Formulation Optimization, Affinity Profiling and Molecular Modeling

Pradeep Kumar, Yahya E. Choonara and Viness Pillay

The presentation will focus on three major aspects of mathematical programming and analysis for formulation optimization, interpretation of the performance of various drug delivery systems, and affinity profiling of various experimental variables and the respective responses:

Design of Experiments: Response Surface Methodology computations for the optimization study are performed employing Minitab ®V15 software. Polynomial models including interaction and quadratic terms are generated for the response variables using Multiple Linear Regression Analysis (MLRA). The general form of the model is represented in Equation (1).

$$Y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \beta_3 X_1 X_2 + \beta_4 X_{12} + \beta_5 X_{22} + \beta_6 X_1 X_{22} + \beta_7 X_{12} X_2 \tag{1}$$

where, β_0 = intercept representing the arithmetic average; β_1 to β_7 are the coefficients computed from the observed experimental values of Y; and X_1 and X_2 are the coded levels of the independent variable(s). The terms X_1X_2 and X_i^2 (i =1 to 2) represent the interaction and quadratic terms, respectively. The optimized combination developed is evaluated for the responses and the experimental values obtained are compared with those predicted by the mathematical models generated.

Artificial Neural Networks: Sensitivity testing and optimization was conducted by employing a feedback Multilayer Perceptron (MLP) neural network to train the empirical input bond-energy data with static back propagation. The MLP is a layered feedforward network typically trained with back propagation of errors using gradient descent or conjugate gradient algorithms. A genetic algorithm with a Sigmoid Axon transfer function and Conjugated Gradient learning rule was employed for the hidden input and output layers.

Molecular mechanics assisted model building and energy refinements: The analytico-mathematical representation of potential energy surfaces, was used to generate the MMER model for potential energy factor as is shown in Eq. (2).

$$E_{molecule/complex} = V_{\sum} = V_b + V_\theta + V_\varphi + V_{ij} + V_{hb} + V_{el} \tag{2}$$

where,$V_{\sum}$ = total steric energy, V_b = bond stretching contributions, V_θ = bond angle contributions, V_φ = torsional contribution, V_{ij} = van der Waals interactions, V_{hb} = hydrogen-bond energy function, and V_{el} = electrostatic energy.

Real-life examples of the application of these techniques will be provided and a unique combinatorial approach of using these mathematical systems under one umbrella will be discussed.

Flow of a Thin Ribbon of Molten Glass in a Tin Bath

David Mason

In the manufacture of glass a thin ribbon of molten glass flows from the furnace onto a bath of liquid tin. Using the thin fluid film approximation a system of partial differential equations is derived for the components of the fluid velocity , pressure and temperature, from the conservation of mass equation and the balance laws for momentum and energy. The viscosity of the molten glass depends on temperature. The equations and boundary conditions are still quite complicated and therefore the approximation of a narrow ribbon is considered. A perturbation expansion in the ratio of the width to the length of the glass ribbon is investigated. In order to complete the zero order system of equations the first order level of approximation has to be considered. Some numerical solutions of the zero order system are derived.

Adaptive Function Approximation for Function Synthesis Using Wavelets

Michael Mitchley

Function approximation methods in reinforcement learning have traditionally faced the curse of dimensionality, with the number of functions or parameters scaling exponentially with dimension. We present a novel mesh-free adaptive function approximation method which uses wavelets (or other refinable equations) to synthesise functions at appropriate levels of detail in arbitrarily high dimensional domains.

Nonclassical Symmetry Solutions for Reaction-Diffusion Equations with Spatially Dependent Diffusivity

Raseelo Moitsheki

Nonclassical and classical symmetry techniques are employed to analyse the reaction-diffusion equations. Here, the diffusivity (diffusion term) is assumed to be an arbitrary function of spatial variable. Direct group classification is attempted. It turns out that the diffusivity needs to be given as a quadratic function of spatial variable for the given governing equation to admit nonclassical symmetries. Both nonclassical and classical symmetries are used to construct some exciting group-invariant (exact). The results are applied to models arising in population dynamics.

Effects of Double Stratification on Unsteady Hydromagnetic Thermal Boundary Layer of Nanofluid Past a Flat Surface

Winifred Mutuku

A similarity solution for the effects of double stratification on heat and mass transfer in an unsteady hydromagnetic boundary layer flow of a nanofluid past a flat surface is presented. The transport equations employed in the analysis include the effects of Brownian motion, thermophoresis, thermal stratification and solutal stratification. The non-linear partial differential equations governing the flow system together with their associated boundary conditions are transformed into a system of nonlinear ordinary differential equations using appropriate similarity variables. A numerical shooting technique with a fourth-order Runge-Kutta-Fehlberg integration scheme was used to solve the resulting boundary value problem. The effects of different pertinent parameters, namely; solutal and thermal stratification, Lewis number, thermophoresis, Brownian motion, magnetic field and unsteadiness on the fluid velocity, temperature, skin friction coefficient, the local Nusse lt number and the local Sherwood number are graphically depicted and quantitatively discussed in detail taking into account the practical applications of the model problem.

Comparison of Different Approaches to Construct First Integrals for Ordinary Differential Equations

Imran Naeem

Different approaches to construct first integrals for ordinary differential equations and systems of ordinary differential equations are studied here. These approaches can be grouped into three categories, Direct methods, Lagrangian or partial Lagrangian formulations and characteristic(multipliers) approaches. The direct method and symmetry conditions on the first integrals correspond to first category. The Lagrangian and partial Lagrangian include three approaches: Noether's theorem, the partial Noether approach and the Noether approach for the equation and its adjoint as a system. The characteristic method, the multiplier approaches and the direct construction formula approach require the integrating factors or characteristics or multipliers. The Hamiltonian version of Noether's theorem is presented to derive first integrals. We apply these different approaches to derive the first integrals of harmonic oscillator equation. We also study first integrals for some physical models. The first integrals for nonlinear jerk equation and the free oscillations of a two degrees of freedom gyroscopic system with quadratic nonlinearities are derived. Moreover solutions via first integrals are also constructed.

Stable, High Order Accurate Adaptive Schemes for Long Time, Highly Intermittent Geophysics Problems

B. A. Erickson and J. Nordström

Many geophysical phenomena are characterized by properties that evolve over a wide range of scales which introduce difficulties when attempting to model these features in one computational method. We have developed a high-order finite difference method for the elastic wave equation that is able to efficiently handle varying temporal and spatial scales in a single, stand-alone framework. We apply this method to earthquake cycle models characterized by extremely long interseismic periods interspersed with abrupt, short periods of dynamic rupture. Through the use of summation-by-parts operators and weak enforcement of boundary conditions we derive a provably stable discretization. Time stepping is achieved through the implicit θ-method which allows us to take large time steps during the intermittent period between earthquakes and adapts appropriately to fully resolve rupture.

An Optimization Model for the Control of Cholera

Jeffrey Ochoche

Infectious diseases continue to be a leading cause of mortality globally. Cholera, once a global disease is now relegated to low and middle income countries thriving in communities with poor hygienic conditions and lack of access to portable water. Even when cholera does not kill it reduces the quality of life and retard economic growth causing undue panic and public concern. In Peru 1991, the estimated cost of cholera outbreak due to food trade embargoes and adverse effect on tourism is US$770 million. Developing countries cannot afford to spend such whooping sums on a preventable disease like cholera, it is therefore important that adequate attention be paid to the control of such diseases. In this paper, we presented a mathematical model for the optimal control of cholera. Seeking the best means of reducing the final size of the epidemic, we introduced three time dependent controls namely, provision of clean water, social distancing and treatment. Using optimal control theory we develop an objective functional to minimize the number of infectives. The Pontryagin's Maximum principle was used to convert the problem of minimization of the objective functional coupled with the state variable into a problem of minimizing point-wisely a Hamiltonian. The optimal control is obtained by solving the optimality system which was composed of four nonlinear ODEs with initial conditions and four nonlinear adjoint ODEs with transversality conditions. The results were analysed and interpreted numerically using MATLAB. The optimal policies identified could have a substantial effect in the control of cholera if implemented. We found that in the case of limited resources, a combination of social distancing and treatment is equally as good as all three policies. Further, a combination of provision of clean water and social distancing is also capable clearing the epidemic but takes more time.

On a Continuous Formulation of Block Method for Solving First Order Ordinary Differential Equations

Abdu Sagir

The aim of this paper is to investigate the performance of the new develop linear multistep block method of uniform order (4,4,4,4)T for the solution of first order initial value problems of ordinary differential equations (ODEs) with associated initial conditions. The method calculates the numerical solution at four points simultaneously and produces four new equally spaced solution values within a block. The continuous hybrid formulation enables us to differentiate and evaluate at some grids and off â?" grid points to obtain four discrete schemes, which were used in block form for parallel or sequential solution of the problems. A stability analysis and efficiency of the block method are tested on ordinary differential equations, and the numerical results obtained compared favourably with the exact solution. Furthermore, comparison of errors has been analysed.

Error Estimates for Finite Element Approximations of a Viscous Wave Equation

Belinda Stapelberg

Karaa (2011) considers a family of fully discrete finite element schemes for solving a viscous wave equation, where the time integration is based on the Newmark method. A rigorous stability analysis based on the energy method is developed. Optimal error estimates in both time and space are obtained. We report on some problems that arose while investigating the article, regarding existence and regularity. Comments are made on some errors and incompleteness of some ideas and statements.

Elastic Waves in a Timoshenko Beam

Alna van der Merwe

Elastic waves in thin structures remains an important topic in structural dynamics. The Timoshenko beam model is as realistic as a one dimensional model can be expected to be. For this model, I discuss numerical simulations of waves that develop in the beam from localised initial disturbances. Interesting features of the elastic waves and changes in the energy distribution are illustrated. The need for considering high frequency modes and the significance of the "second spectrum" of the beam model in understanding the properties of the elastic waves are explored. Finally, considering the properties of the elastic waves provides insight into the effect of boundary damping on the waves.

Galerkin Finite Element Approximation of General Linear Second Order Hyperbolic Equations

Nic van Rensburg

We derive error estimates for the Galerkin approximation of a general second order equation. The results can be applied to a variety of cases e.g. vibrating systems of linked elastic bodies. The results generalize the work of Baker (1976) and allow also for viscous type damping. Splitting the proofs for the semidiscrete and fully discrete cases not only simplifies the proofs but less restrictive regularity assumptions are required.

Domain Splitting for Chebyshev Spectral Collocation

André Weideman

Polynomial interpolants at Chebyshev points converge exponentially fast when the function that is approximated is analytic in an open neighbourhood of the interval of interest, say [-1,1]. This has been the basis of the success of the Chebyshev spectral collocation method for solving differential equations, as well as the Chebfun software system. When the domain of analyticity is very narrow, i.e., when there is a singularity in the complex plane close to the real interval [-1,1], the convergence can be rather slow, however. In such cases the spectral method converges not much faster, if at all, than low-order approximation methods such as finite differences or finite elements. In this talk we examine the possibility of improving the convergence rate by splitting the interval into two, and then applying two separate interpolants on each subinterval, maintaining appropriate continuity across the splitting point. The optimal splitting point is determined by the location of the singularity in the complex plane closest to [-1,1], and we show how to derive an explicit formula for the splitting point. We also derive a critical ellipse in the complex plane that encloses [-1,1], with the property that if the singularity lies inside this ellipse, then splitting is advantageous, otherwise it is not. A numerical experiment involving a two-point boundary value problem with an internal layer will be presented.

Computational Approaches In Compressed Sensing

Matthew Woolway

This presentation aims to provide a summary on computational approaches to solving the Compressed Sensing problem. The theoretical problem of solving systems of linear equations has long been investigated in academic literature. A relatively new field, Compressed Sensing is an application of such a problem. Specifically, with the ability to change the way in which we obtain and process signals. Under the assumption of sparse signals, Compressed Sensing is able to recover signals much lower than that of the current Shannon/Nyquist theory. The primary goal of this thesis, is to describe major algorithms currently used in the Compressed Sensing problem. This is done as a means to provide the reader with sufficient up to date knowledge on current approaches as well as their means of implementation, on central processing units (CPUs) and graphical processing units (GPUs), when considering computational concerns such as computational time, storage requirements and parallelizability.

Modification of First Order ODEs Numerical Method for Solution of nth Order ODEs

Ayobami Zamurat

An illustration of how any first order ordinary differential equations (ODEs) numerical method can be modified for a higher nth order ODEs was presented. The theory of Nystöm method was adopted in the modification of the methods. Through this process a first order ODEs numerical method can be extended to the case in which the approximate solution to a higher nth order ODEs (special or general), as well as first order Initial Value Problems (IVPs) can be calculated. Also the convergence and stability analysis the methods were presented. Numerical examples are given to illustrate the modification.

Monday 14th April

Middle Room

08:00 - 09:00	Registration
09:00 - 09:30	Welcome Ceremony
Chair: E. Momoniat	
09:30 - 10:30	**Plenary: J. Nordström:** *High Order Finite Difference Approximations of Multi-physics Problems*
10:30 - 11:00	Tea Break
Chair: J. King	
11:00 - 11:30	**M. Basson:** *Galerkin Finite Element Approximation of General Linear Second Order Hyperbolic Equations with General Damping*
11:30 - 12:00	**B. Stapelberg:** *Error Estimates for Finite Element Approximations of a Viscous Wave Equation*
12:30 - 13:30	Lunch Break
Chair: J. Nordström	
13:30 - 14:30	**Plenary: L. Reichel:** *Network Analysis via Gauss-Type Quadrature and Partial Spectral Factorization*
Chair: C. Harley	
14:30 - 15:00	**A. Zamurat:** *Modification of First Order ODEs Numerical Method for Solution of nth Order ODEs*
15:00 - 15:30	**R. Kareem:** *Numerical Solutions of stiff Initial Value Problems in ODEs Using Modified Extended Backward Differentiation Formula*
Chair: L. Reichel	
15:30 - 16:30	**Plenary: B. Henry:** *Stochastic Models for Diffusive Transport with Traps, Reactions and Forces*

East Room

09:30 - 10:30	Plenary
10:30 - 11:00	Tea Break
Chair: R. S. Herbst	
11:00 - 11:30	**W. Mutuku:** *Effects of Double Stratification on Unsteady Hydromagnetic Thermal Boundary Layer of Nanofluid Past a Flat Surface*
11:30 - 12:00	**A. Haywood:** *Particle Occlusion in Gold Nanoparticles*
12:30 - 13:30	Lunch Break
13:30 - 14:30	Plenary
Chair: G. Fareo	
14:30 - 15:00	**P. Kumar:** *In Silico Mathematical Analysis and Programming for the Performance Interpretation of Drug Eluting Assemblies: Formulation Optimization, Affinity Profiling and Molecular Modeling*
15:00 - 15:30	**J. Ochoche:** *An Optimization Model for the Control of Cholera*
15:30 - 16:30	Plenary

Tuesday 15th April

Middle Room

Chair: B. Henry	
09:00 - 10:00	**Plenary: J. King:** *Self-Similar Behaviour in Nonlinear Diffusion*
10:00 - 10:30	Tea Break

Chair: A. Hutchinson	
10:30 - 11:00	**D. Mason:** *Flow of a Thin Ribbon of Molten Glass in a Tin Bath*
11:00 - 11:30	**G. Fareo:** *Approximate Solutions to the Problem of a Pre-existing Fracture Driven by a Power-law Fluid*
11:30 - 12:00	**R. Kgatle:** *Propagation of a Linear Hydraulic Fracture with Tortuosity*
12:00 - 12:30	**A. v d Merwe:** *Elastic Waves in a Timoshenko Beam*
12:30 - 14:30	Lunch Break

Chair: D. Mason	
14:30 - 15:00	**R. J. Moitsheki:** *Nonclassical Symmetry Solutions for Reaction-diffusion Equations with Spatially Dependent Diffusivity*
15:00 - 15:30	**I. Naeem:** *Comparison of Different Approaches to Construct First Integrals for Ordinary Differential Equations*
15:30 - 16:00	**H. Gidey:** *Time-Splitting Procedures for the Numerical Solution of the 2D Advection-Diffusion Equation*

East Room

09:00 - 10:00	Plenary
10:00 - 10:30	Tea Break

Chair: B. Jacobs	
10:30 - 11:00	**A. Weideman:** *Domain Splitting for Chebyshev Spectral Collocation*
11:00 - 11:30	**M. Mitchley:** *Adaptive Function Approximation for Function Synthesis Using Wavelets*
11:30 - 12:00	**I. Fabris-Rotelli:** *Compressed Sensing*
12:00 - 12:30	**M. Woolway:** *Computational Approaches In Compressed Sensing*
12:30 - 14:30	Lunch Break

Chair: C. Harley	
14:30 - 15:00	**G. Cooper:** *Determining the Depth and Location of Dykes from Aeromagnetic Data*
15:00 - 15:30	**J. Nordström:** *Stable, High Order Accurate Adaptive Schemes for Long Time, Highly Intermittent Geophysics Problems*
15:30 - 16:00	**S. du Toit:** *Finite Element Analysis of the Vibration of a Vertical Slender Structure on a Resilient Seating*

Wednesday 16th April

Middle Room

Chair: W. Hereman	
09:00 - 10:00	**Plenary: N. Hale:** *Chebfun and Related Research*
10:00 - 10:30	Tea Break
Chair: R. J. Moitsheki	
10:30 - 11:00	**N. van Rensburg:** *Galerkin Finite Element Approximation of General Linear Second Order Hyperbolic Equations*
11:00 - 11:30	**S. Childs:** *The Finite Element Implementation of a K.P.P. Equation for the Simulation of Tsetse Control Measures in the Vicinity of a Game Reserve*
11:30 - 12:00	**B. Jacobs:** *A New Grünwald-Letnikov Derivative Derived From a Second-Order Scheme*
Chair: N. Hale	
12:00 - 13:00	**Plenary: W. Hereman:** *Symbolic Computation of Conservation Laws of Nonlinear Partial Differential Equations*
13:00 - 14:00	Lunch Break

East Room

09:00 - 10:00	Plenary
10:00 - 10:30	Tea Break
Chair: H. Gidey	
10:30 - 11:00	**R. S. Herbst:** *Numerical Simulation of Isothermal Gravitational Collapse*
11:00 - 11:30	**A. Badmus:** *Hybrid Implicit Block Algorithms for Improved Performance In The Solution Of First Order Initial Value Problems*
11:30 - 12:00	**A. N. Sagir:** *On a Continuous Formulation of Block Method for Solving First Order Ordinary Differential Equations*
12:00 - 13:00	Plenary
13:00 - 14:00	Lunch Break